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STUDY ON CHARACTERIZATION OF HORIZONTAL CRACKS IN ISOTROPIC BEAMS

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A theoretical analysis and experimental study to characterize horizontal crack in isotropic beams is presented. In the theoretical analysis, the beam is divided into domains and a harmonic load is applied on its surface. Displacement in the thickness direction along the beam was measured. Strip element method (SEM) was also used to compute these displacements for beams under study. Comparisons between theoretical and experimental results are then conducted. Both results show that the crack length can be determined from the response pattern. It is also found that higher-frequency excitation is needed to obtain a clear indication of the location of deeper cracks. The results from this study show that the present technique is an efficient alternative method to characterize horizontal crack in isotropic beams.

1. INTRODUCTION

The identification of cracked beam has become the subject of many research works. Many non-destructive testing (NDT) methods have been developed. These methods have their own advantages and disadvantages. The consumption of excess time is one very common disadvantage among existing methods. Therefore, a fast and simple NDT method is a very attractive proposition.

In order to accomplish the above requirement, several studies on non-destructive testing using vibration technique in elastic material have been reported. Related to the experimental study, Adams *et al.* [1] found that a state of crack could be detected by a reduction in stiffness and an increase in damping. Changes in stiffness lead to changes in the natural frequency of a vibrating system. Therefore, it was suggested that the measurement of the natural frequency offers the possibility of locating crack in a structure and determines the severity of the crack. This was followed by Rizos *et al.* [2] who used the measured amplitudes at two points of the vibrating structure at one of its natural modes to find the location and estimate the depth of the crack in a cantilever beam. Narkis [3] conducted numerical finite element calculations and natural frequency measurement to identify the crack location in a simply supported beam. Boltezar *et al.* [4] proposed an experimental technique based on the measurement of the axial and flexural vibration response to identify the existence of crack in a specimen.

In theoretical analysis, Liu and Achenbach [5] have used the strip element method (SEM) to investigate the wave scattering by crack in anisotropic laminated plates. In solving the wave-scattering problem, the SEM needs a much smaller number of equations compared to the FEM. Initially, the SEM was proposed by Liu and Achenbach [6] for stress analysis of anisotropic linearly elastic solid. In this method, the specimen is divided

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into a set of strip elements. A dimension-reduced system of approximate differential equations for the strip elements is constructed using the principle of virtual work. This method requires less data storage compared to FEM, but maintains many advantages of the FEM and BEM. Liu and Lam [7, 8] had also used the SEM for characterization of horizontal and vertical cracks in anisotropic laminated plates. These theoretical studies have found that techniques using flexural waves can be a very promising alternative means for detecting and characterizing cracks in beams. In contrast to the conventional ultrasonic method which employs high-frequency signal, the flexural wave technique is conducted using low-frequency signal. Hence, the testing is fast and does not require coupling fluids. The important thing in applying the flexural wave technique is to determine the effect of crack on the characteristics of the wave motion in the beam.

In this paper, an NDT technique using flexural waves is investigated theoretically and experimentally. SEM is used to calculate the scattering of the wave fields by a horizontal crack in an isotropic beam. An experimental study using an FFT analyzer, an electromagnetic exciter and an accelerometer is then conducted on several specimens with simulated cracks. Both theoretical and experimental results indicate the crack can be detected through the beam's response.

2. THE STRIP ELEMENT METHOD

2.1. DESCRIPTION OF THE PROBLEM

Consider an infinite isotropic beam with a thickness H and a horizontal crack as shown in Figure 1. One assumes that the beam lies on the region $-\infty\,(x, y) \leqslant \infty,\ -H \leqslant z \leqslant 0$. The length of the horizontal crack and the depth of the crack from the upper surface of the specimen are represented as a_c and d_c respectively. In order to simplify the problem, one assumes that the crack covers the whole width of the beam and the harmonic loading is uniform over the width in the y direction. The analytical study now reduces to a two-dimensional case. The excitation force acting on the upper surface of the beam is a time harmonic load fixed at $x = 0$ and can be represented as

$$F(x, t) = f_0 \exp(\mathrm{i}\omega t), \tag{1}$$

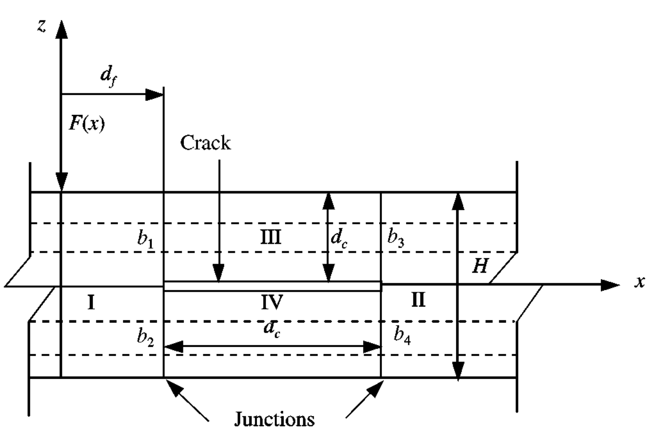

Figure 1. Division of beam with horizontal crack.

where f_0 is the amplitude of the sinusoidal force. It is expected that the excited flexural waves in the beam will be scattered by the crack, and carry the information regarding the presence and the geometry of the crack.

2.2. THE PROCEDURE OF STRIP ELEMENT METHOD

The procedures for using the SEM to solve the two-dimensional problem of wave propagation in an isotropic beam can be presented as follow:

(1) Divide the beam into some strip elements in the thickness direction (z direction).

(2) Derive an approximate differential equation for the field dependencies by using the principle of virtual work for each element. This equation can be expressed as

$$\rho \ddot{\mathbf{U}} - \mathbf{L}^{\mathrm{T}} \mathbf{c} \mathbf{U} = 0, \tag{2}$$

where $\mathbf{U}$ represents the displacement vector, ρ is the density of the material, and $\mathbf{L}$ is a matrix of differential operator expressed by

$$\mathbf{L}^{\mathrm{T}} = \begin{bmatrix} \dfrac{\partial}{\partial x} & 0 & \dfrac{\partial}{\partial z} \\ 0 & \dfrac{\partial}{\partial z} & \dfrac{\partial}{\partial x} \end{bmatrix}. \tag{3}$$

In Equation (2) the matrix $\mathbf{c} = (c_{ij})$, $i, j = 1, 2, 3$ represents the elastic constants. In equations (2) and (3), the superscript “T” denotes the transpose matrix.

(3) In each element, the displacement vector $\mathbf{U}_e^{\mathrm{T}}(x, z) = \{u(x, z)\, w(x, z)\}$, where $u(x, z)$ and $w(x, z)$ are the displacement components in the x and z directions respectively. The displacement function is expressed as a product of the interpolation function N and the displacement function V_e.

$$\mathbf{U}_e(x, z, t) = \mathbf{N}(z)\mathbf{V}(x)\exp(\mathrm{i}\omega t) \tag{4}$$

where the variables t and ω are time and angular frequency respectively.

(4) Applying the principle of virtual work to a strip element, a set of approximate differential equations can be derived for the displacement over the lines connecting the node point. Assemble the approximate differential equation for all elements by using the boundary conditions in the horizontal direction. Finally, a set of second order governing differential equations for the specimen can be obtained as follows:

$$\mathbf{f} = \left[-\mathbf{A}_{2t}\frac{\partial^2 V}{\partial x^2} + \mathbf{A}_{1t}\frac{\partial V}{\partial x} + \mathbf{A}_{0t} - \omega^2 \mathbf{M}_t \mathbf{V} \right], \tag{5}$$

where $\mathbf{f}$ represents the external force vector acting on the node lines, and the matrices $\mathbf{A}_{it}$, $\mathbf{M}_t$ and vector $\mathbf{V}$ can be obtained by assembling the corresponding matrices $\mathbf{A}_i$, $\mathbf{M}$ and vector $\mathbf{V}_e$ of adjacent elements. The matrices $\mathbf{A}_{it}$ and $\mathbf{M}_t$ for isotropic materials can be found in the appendices of the paper by Liu and Achenbach [6].

(5) Solve the set of approximate differential equations analytically to obtain a particular solution and the complementary solution.

(6) Replace the unknown constants in the complementary solution by unknown displacement on the vertical boundaries. This calculation will produce a set of equations, which gives the relationship between the displacements and external traction at the node point on the vertical boundaries. This equation can be expressed as

$$\mathbf{R}_b = \mathbf{K}\mathbf{V}_b + \mathbf{S}_b, \tag{6}$$

where **K** is the stiffness matrix, $\mathbf{S}_b$ is the equivalent external force vector acting on the vertical boundary, $\mathbf{R}_b$ and $\mathbf{V}_b$ are the external traction and displacement vectors on the vertical boundaries.

(7) Solve the set of equations by imposing the vertical boundary conditions. The whole wave field can be obtained.

More detailed formulations about this method have been explained in papers by Liu and Achenbach [5, 6].

3. EXPERIMENTAL STUDY

A series of experimental investigations was carried out on two types of specimens. Type 1 specimen consists of two crack configurations (A and B), and type 2 specimen consists of three different crack configurations (A, B and C). The dimensions and configurations of these specimens are shown in Figure 2.

Specimen 1 is made from perspex of 800 mm length, 20 mm width and 5 mm thickness. It is considered as a plane-strain beam because the thickness in the z direction is much smaller when compared with the dimension of the other two directions. Unlike specimen 1A that has no crack, specimen 1B has a simulated crack in its thickness direction. The parameters for the crack are $a_c = 25$ mm and $d_c = 2{\cdot}25$ mm.

Specimen 2 is made from perspex of 800 mm length, 5 mm width and 20 mm thickness. It can be considered as a plane-stress beam because the thickness in the z direction is much larger than the thickness in the y direction. The crack of specimen 2 is 25 mm long and at a distance of 482·5 mm from the left end of the beam. The depth of the crack of specimens 2A, 2B and 2C is 4·75, 9·75 and 14·75 mm, respectively, from the surface. The crack is created using a very small milling cutter of 0·25 mm diameter. The manufacturing process creates a 0·5 mm gap on the crack.

Two hundred millimeters of the beam at each end is immersed in sand, as shown in Figure 2 by the shaded areas. It is done to minimize reflection at the boundaries and to simulate an infinite beam. In a real situation, the beam will have arbitrary boundary conditions. However, the experimental procedure can still be applied if the measuring point is far away from the boundary where the effect of reflection is minimized. A schematic drawing of the experimental set-up is shown in Figure 3.

The beam's response along its surface was measured using a Kistler accelerometer (type 8614A). The accelerometer was attached to the measurement points using a *cementing stud*. Sweep sine signal with a frequency range from 0 to 20 kHz was generated by an LDS oscillator type TPO 25 and applied to the specimen via an LDS electro-dynamic exciter type V101 connected to a coupling rod. In this experimental study, the excitation point was fixed but the response was measured along the beam surface with an interval of 10 mm. For the points at the crack region, measurement was conducted with an interval of 5 mm. In total, 40 measurements were recorded. The measured response signal from the accelerometer was then amplified using a power coupler. Subsequently, the output signals from the power coupler were recorded and analyzed using a Hewlett Packard-type 35670A dynamic signal analyzer. A response curve can be obtained by plotting the response against x, which is the distance between the excitation and the measurement point.

The sandbox is tapered for a smoother transition in the impedance. This is to ensure that the outgoing flexural waves will be damped gradually without reflection. As a result, a non-reflecting boundary or an infinite beam can be experimentally simulated.

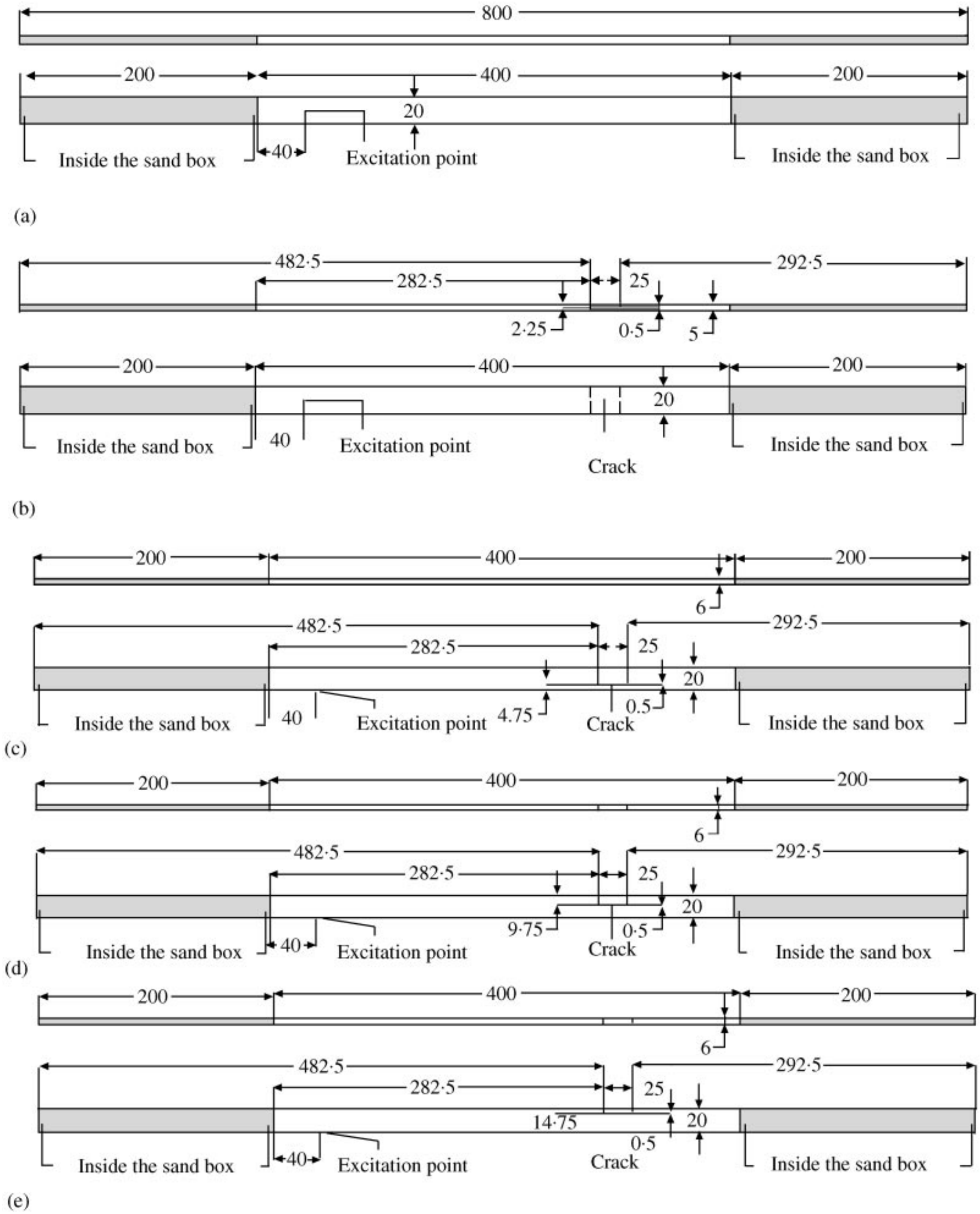

Figure 2. Specimens used in the experimental study (unit: mm): (a) Specimen 1A (without crack), (b) Specimen 1B (with crack of 2·25 depth), (c) Specimen 2A (with crack of 4·75 mm depth), (d) Specimen 2B (with crack of 9·75 mm depth), (e) Specimen 2C (with crack of 14·75 mm depth).

4. COMPARISON STUDY

4.1. A TECHNIQUE TO DETERMINE HORIZONTAL CRACKS

Before comparing the theoretical and experimental results, one has to arrange both the results to ensure that they are ready to be compared. The SEM results represent the beam's

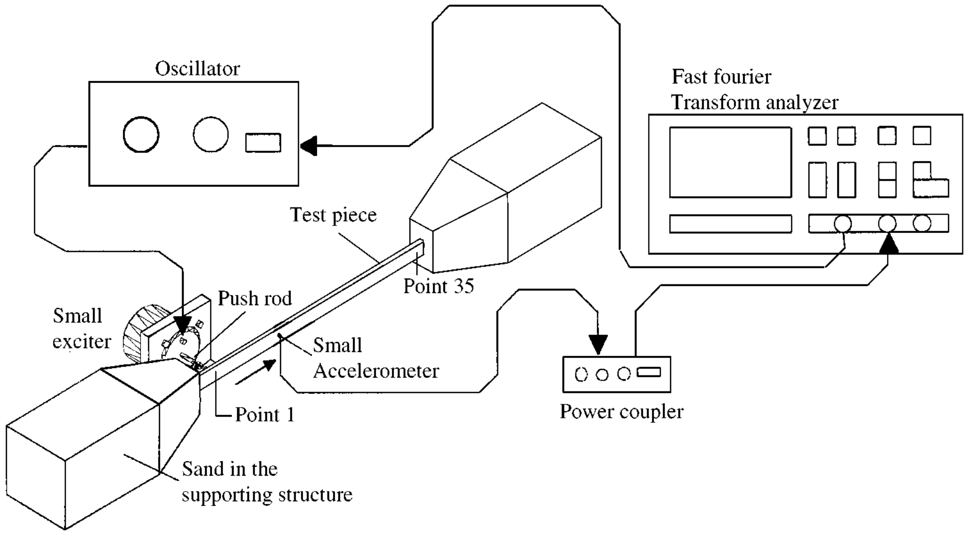

Figure 3. Schematic drawing of the experimental set-up.

displacement along its surface. The raw experimental data represent the surface beam's response at different excitation frequencies.

On experimental data, firstly one has to compile all the measured beam's response. By presenting the beam's response along the measuring point in a given frequency, one will find the variation of the beam's response over its surface. To get a dimensionless response, one normalizes both sets of data to the highest value of each set.

The FEM was used to investigate the effect of a 0·5 mm crack gap to the result of the theoretical analysis [9]. It has been confirmed that the effect of the gap is very small. Therefore, in this paper the theoretical results are obtained using SEM without considering the gap.

For beam without crack, its response is a smooth harmonic curve with no significant change over the whole length. However, for beam with crack, its response may have a significant change when the wave passed through the crack area. The length of the region where the pattern changes can be used to approximately determine the crack length. Moreover, the frequency in which the clear response was detected can be expected to contain information related to the crack depth.

4.2. DETECTION OF CRACK IN ISOTROPIC BEAM

4.2.1. *Crack length*

The calculated and measured values for response (acceleration) of specimen 1A along its surface at frequencies of 6515 and 6666 Hz are shown in Figures 4(a) and 4(b) respectively. The calculated and measured results have been normalized.

Figure 4(a) shows that the SEM result agrees reasonably well with the experimental result. Both curves show that there is no pattern change in the beam's response along its length. Similar findings can also be observed from Figure 4(b) that is obtained at a higher frequency. On specimen 1A, there is no crack that may reflect or scatter the incident wave. Hence, the wave will propagate smoothly. The different number of peaks observed in

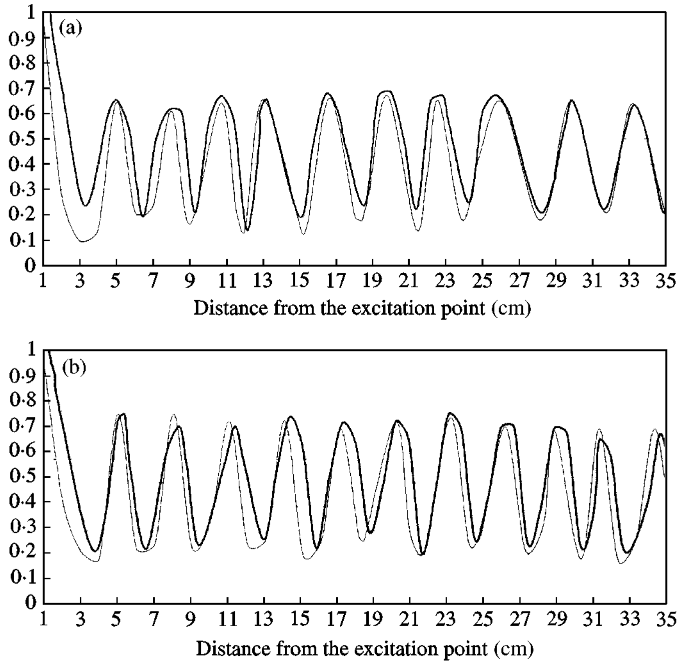

Figure 4. Calculated and measured response of beam 1A: (a) specimen 1A at frequency 6516 Hz, (b) specimen 1A at frequency 6666 Hz. ——, theoretical; ——, experimental.

Figures 4(a) and 4(b) indicate that the beam's responses to the harmonic load are dependent on the excitation frequency.

The calculated and measured responses on the surface of specimen 1B are shown in Figure 5. It is seen again that the SEM results agree reasonably with the experimental results for a beam with crack. From Figure 5 it is noted that the crack length can be approximately determined from the pattern change of the beam response when the wave passes through the crack. In Figure 5, one observes clearly that the starting point of the decaying response occurs at $x = 26{\cdot}5$ cm which corresponds to the right tip of the crack. However, for this case, one cannot observe clearly the left tip of the crack. This can be easily overcome by changing the location of excitation to the right-hand side of the crack. By doing so, one could detect the left and right tips of the crack.

The calculated and measured responses of specimen 2A along its surface are shown in Figure 6. Compared with the response of beam 1B, the response of specimen 2A in Figure 6 shows more clearly the crack region. An increase in the beam's response at $x = 24$ cm indicates the beginning of the crack region. The beam's peak response value is at $x = 25{\cdot}5$ cm, very close to the center of the crack region. After this, the response starts to decline and decay at $x = 26{\cdot}5$ cm. From the curves in Figure 6, one can conclude that the crack lies in between $x = 24$ and $26{\cdot}5$ cm.

4.2.2. *Effects of crack depth*

In the further investigation, one correlates the crack depth with the frequency that gives a clear response. For that, the SEM and experimental study are employed on specimens 2A, 2B and 2C that have, respectively, 4·75, 9·75 and 14·75 mm crack depth from the surface and

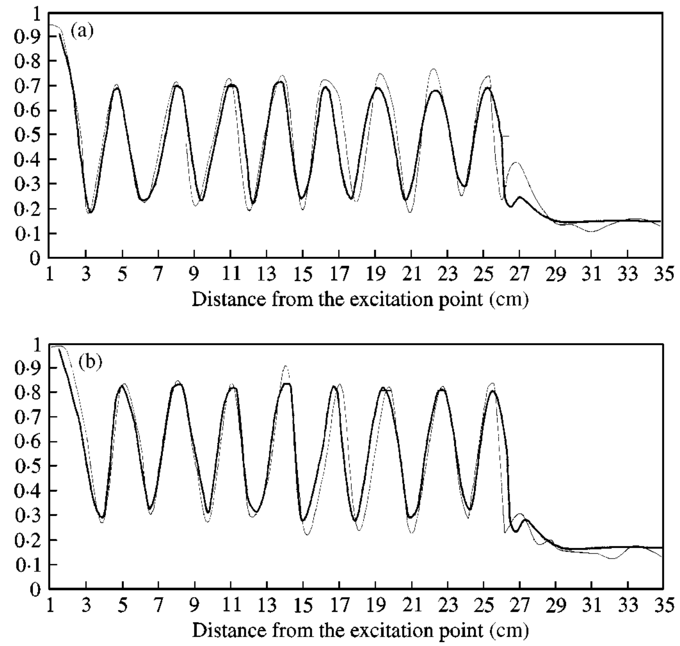

Figure 5. Calculated and measured response of beam 1B: (a) specimen 1B at frequency 6516 Hz, (b) specimen 1B at frequency 6666 Hz. ——, theoretical; ——, experimental.

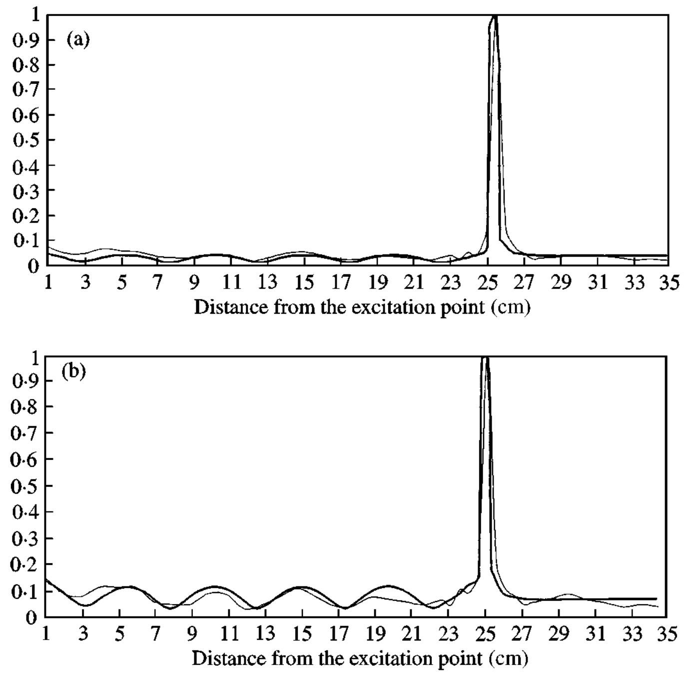

Figure 6. Calculated and measured response of beam 2A: (a) specimen 2A at frequency 6515 Hz, (b) specimen 2A at frequency 6666 Hz. ——, theoretical; ——, experimental.

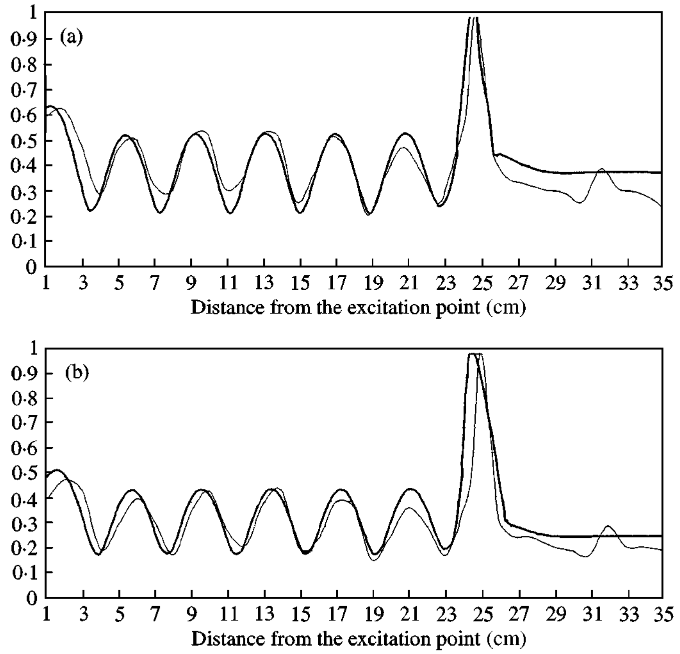

Figure 7. Calculated and measured response of beam 2B: (a) specimen 2B at frequency 10 000 Hz, (b) specimen 2B at frequency 10 250 Hz. ——, theoretical; ——, experimental.

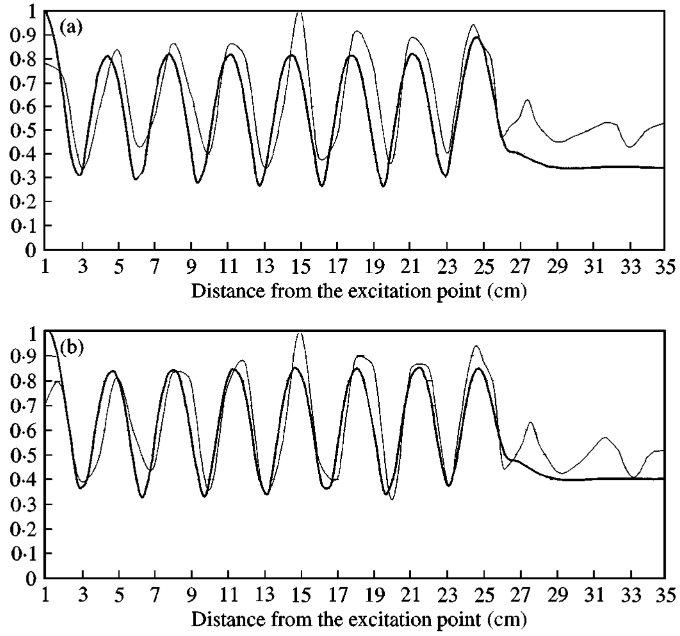

Figure 8. Calculated and measured response of beam 2C: (a) specimen 2C at frequency 12 250 Hz, (b) specimen 2C at frequency 12 500 Hz. ——, theoretical; ——, experimental.

a constant crack length of 25 mm. The calculated and measured responses of specimens 2B and 2C are shown in Figures 7 and 8.

As in the response of specimen 2A, the response of specimen 2B in Figure 7 shows a clear indication of the starting and ending points of the crack region. The response starts to

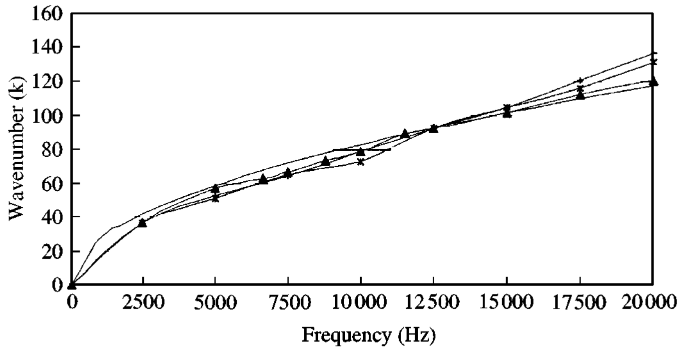

Figure 9. Spectrum relation for perspex beam: ——, theoretical no crack; —▲—, experimental 5 mm crack; —✕—, experimental 10 mm crack; —+—, experimental 15 mm crack.

increase at $x = 24$ cm, has a peak near $x = 25{\cdot}5$ cm, and then decreases to the minimum value at $x = 26{\cdot}5$ cm before decaying.

The response of specimen 2C shows the same pattern as specimen 1B in which only the ending point of the crack region is clearly shown. In this case, the crack is deeper from the measuring surface but closer to the other surface. Thus, if the pattern change is distinct, it can be improved by measuring the response of the beam from the other side.

The responses of specimens 2A, 2B and 2C indicate that in order to detect a deeper crack, one needs to excite the beam with a higher frequency. For detecting a 5 mm deep crack, one needs an excitation frequency of 6515 Hz, for a 10 mm deep crack, one needs a frequency of 10 000 Hz, and for a 15 mm deep crack, 12 250 Hz excitation is needed. It is possible to construct a regression equation that relates the crack depth and the excitation frequency to detect it with sufficient experimental data. The constructed regression equation should eliminate the effect of beam dimension and material properties, and hence can be applied to general beam. From the experimental study, one obtains the excitation frequency to detect crack and using the developed regression equation one can predict the depth of the crack. Further investigation is still being conducted to develop the regression equation.

4.2.3. *Wave analysis*

By analyzing the angular frequency and relating it with the wave speed and wave number, one could define the type of the wave that propagates in the beam. From the experimental study, one obtains the frequency and wavelength data. Employing basic wave equations, one can compute the corresponding wave number and wave speed. The relationship between excitation frequency and wave speed, which is obtained from the experimental study, can be represented very well by the flexural wave equation as follows [10–13]:

$$c = \frac{\omega}{k} = \omega\left(\frac{\omega^2 \rho A - \mathrm{i}\omega\eta A}{EI}\right)^{-1/4}, \tag{7}$$

where c is the wave speed, and k is the wave number. η is the materials damping constant, A is the cross-sectional area of the beam, E, I, and ρ are the Young's modulus, the moment inertia and the mass density of the material respectively. This finding indicates that the flexural wave plays an essential part in detecting the crack in beams. Considering only the propagating wave mode and assuming small damping from the material, the wave speed expression can be simplified as follows:

$$c = \sqrt{\omega}\left[\frac{EI}{\rho A}\right]^{1/4}. \tag{8}$$

In this analysis, the wave speed is presented in dimensionless form by dividing its value by the transversal wave speed, c_2. The calculated and measured data of the first anti-symmetric spectrum relation for perspex beam with 5, 10 and 15 mm depth of crack are shown in Figure 9. Figure 9 shows that the crack depth does not vary the wave number. This spectrum also indicates that the waves propagating in the beam are dispersive.

5. CONCLUSIONS

Analytical study using SEM and experimental study using flexural wave technique have been employed to characterize horizontal crack in isotropic beams. It has been shown that the results from these two methods agree reasonably well. Both the techniques show that the presence of the crack may reflect or scatter the incident wave and generate a very significant change in the pattern of the beam's response. It also shows that the techniques presented can be used to approximately detect the crack depth and accurately determine the crack length in isotropic beams. These two techniques are expected to be both accurate and efficient methods to be used in the characterization of the crack in isotropic beams.

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